Efficient Multi-Objective Optimization for Large-Scale Conservation Planning

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Presentation outline

1. Multiobjective optimization
2. Pareto optimality
3. Multi-objective model in conservation planning
4. Epsilon-constraint method
5. Algorithmic framework
6. Results

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Single-objective optimization problem

$$\max / \min f(x)$$

subject to $x \in X$

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Multiobjective optimization problem

$$\max / \min F(x) = \{f_1(x), f_2(x), \ldots, f_m(x)\}$$

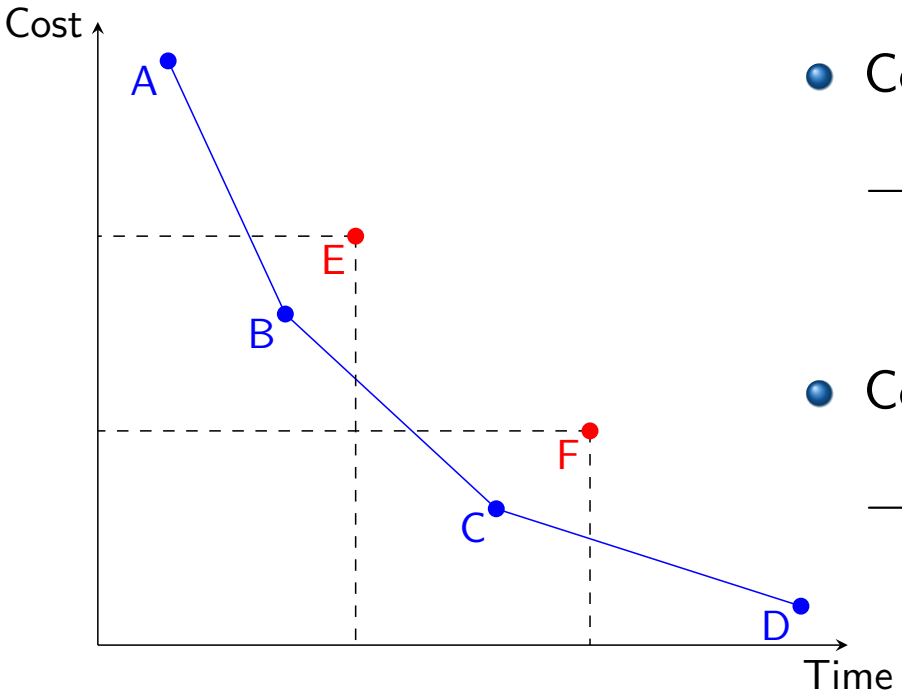
subject to $x \in X$

- x: Decision variables
- $f(x)$: Objective function
- $f_i(x)$: Objective function components
- X: Feasible region

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Dominated and nondominated points



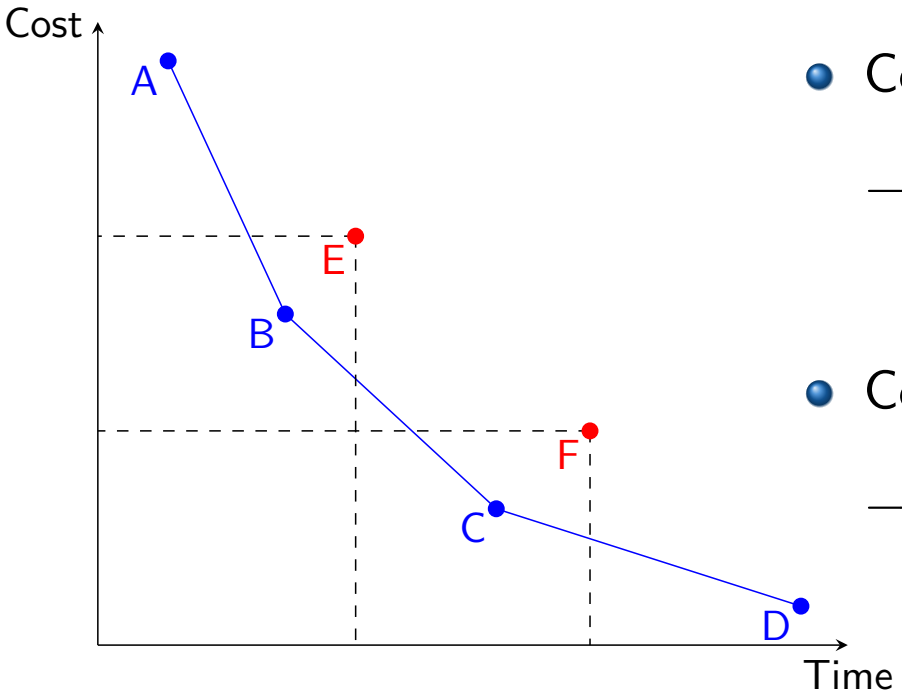
- Cost(B) $<$ Cost(E) & Time(B) $<$ Time(E)

  $\longrightarrow$ B dominates E (or E is dominated by B)

- Cost(C) $<$ Cost(F) & Time(C) $<$ Time(F)

  $\longrightarrow$ C dominates F

Dominated and nondominated points



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  $\longrightarrow$ B dominates E (or E is dominated by B)

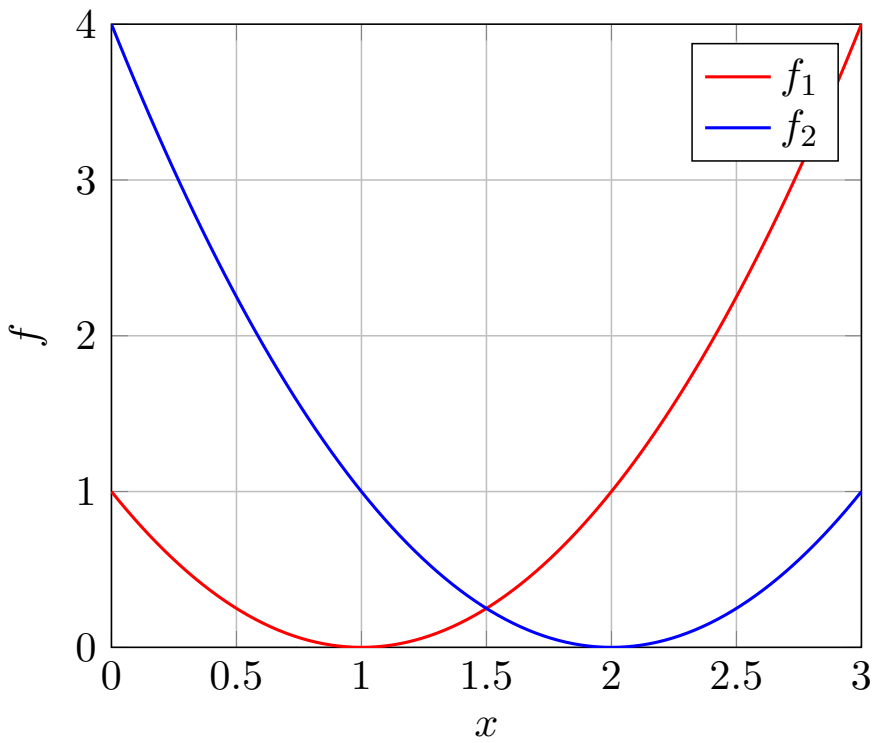
- Cost(C) $<$ Cost(F) & Time(C) $<$ Time(F)

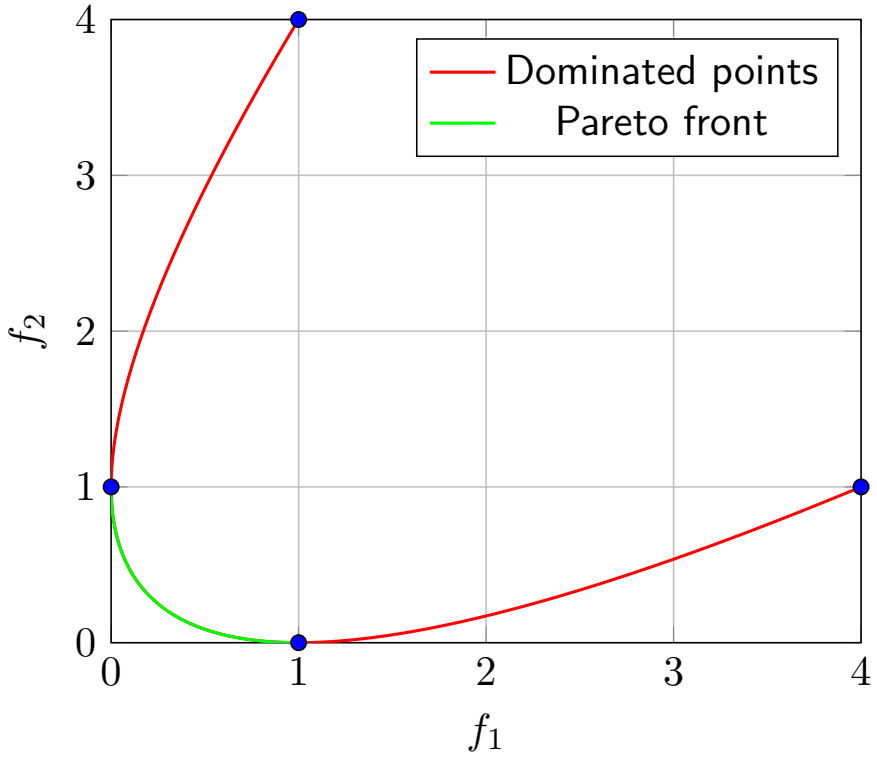
  $\longrightarrow$ C dominates F

- Nondominated points: A, B, C and D
- Dominated points: E and F
- Dominated points can not be solutions

Pareto front example

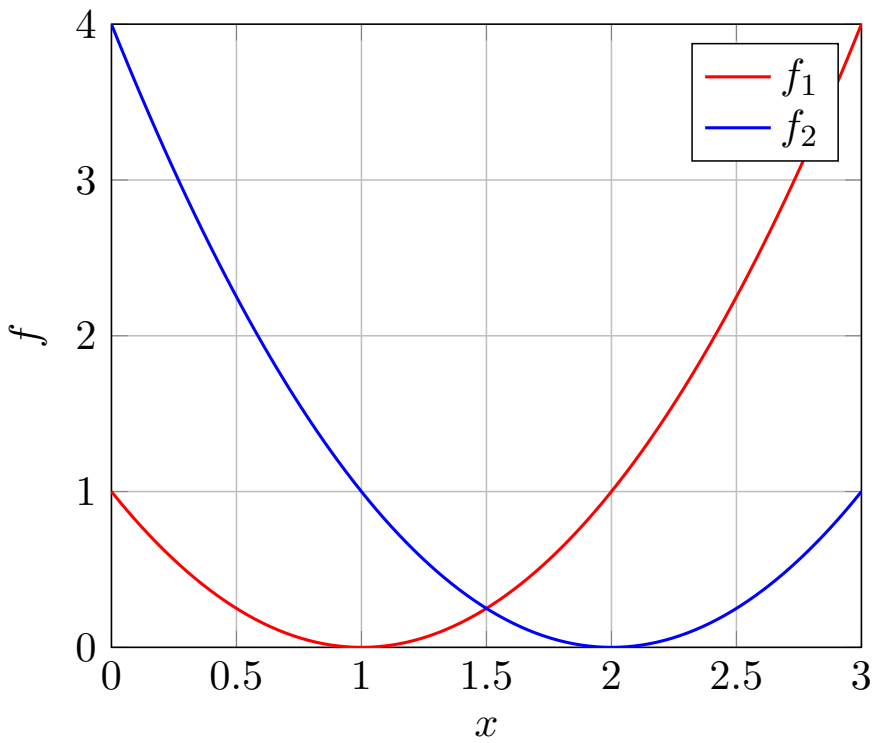
$$\min f_1(x) = (x-1)^2$$
$$\min f_2(x) = (x-2)^2$$
$$\text{s.t. } 0 \leq x \leq 3$$

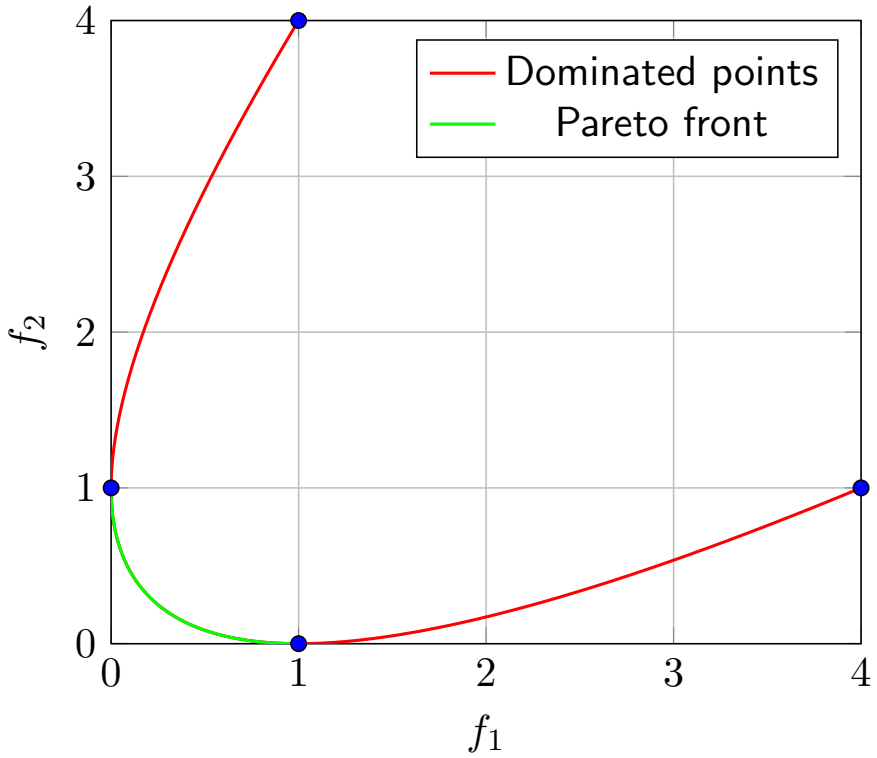




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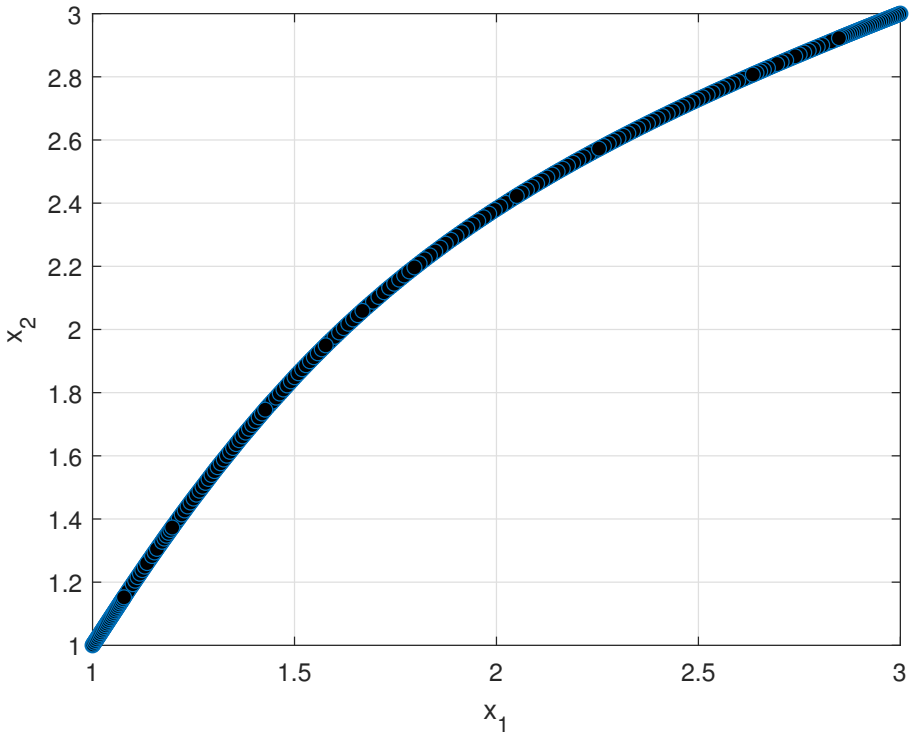


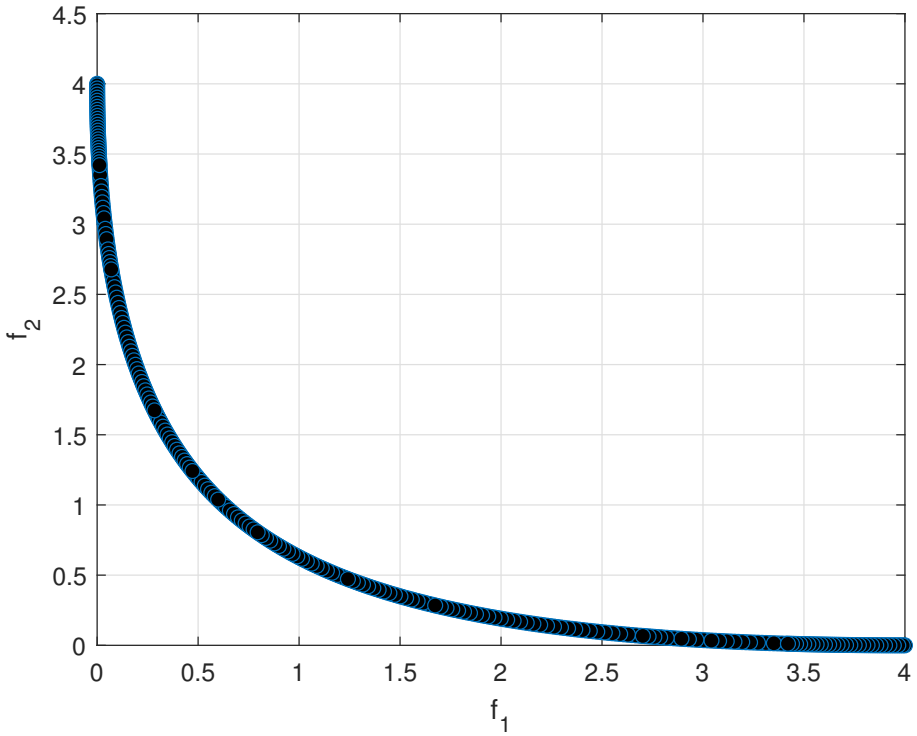


- **Pareto front**:
  - Set of nondominated points in feasible region

Pareto front example

$$\begin{aligned}
\min\ & f_1(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 \\
\min\ & f_2(x) = (x_2 - 3)^2 + (x_1 - x_2)^2 \\
\text{s.t.}\ & -1 \leq x_1 \leq 5 \\
& -1 \leq x_2 \leq 5
\end{aligned}$$





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Multi-objective model in conservation planning

I: Set of planning units (indexed by i)
J: Set of species (indexed by j)

Parameters:

c_i: Cost of planning unit i

D_{ij}: Nucleotide diversity of planning unit i for species j

L_{ijk}: Presence/absence of lineage k for species j in planning unit i

n_j: Number of lineages for species j

A_j: Target level of nucleotide diversity for species j

B_{jk}: Target level of occurrences for lineage k of species j

B_c: Total cost-budget

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Decision Variables:

x_i: Binary decision variable indicating whether planning unit $i \in I$ is selected

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Decision Variables:

x_i: Binary decision variable indicating whether planning unit $i \in I$ is selected

y: The percentage of target shortfalls for nucleotide diversity

w: The percentage of target shortfalls for lineage occurrence

Multi-objective model in conservation planning

Objective Functions:

Minimize: y

Minimize: w

Multi-objective model in conservation planning

Objective Functions:

Minimize: y

Minimize: w

Constraints:

$$\sum_{i \in I} D_{ij}\, x_i + A_j\, y \geq A_j, \quad \forall j \in J$$

$$\sum_{i \in I} L_{ijk}\, x_i + B_{jk}\, w \geq B_{jk}, \quad \forall j \in J,\ \forall k \in \{1, \ldots, n_j\}$$

$$\sum_{i \in I} c_i\, x_i \leq B_c \quad \forall i \in I$$

$$x_i \in \{0, 1\} \quad \forall i \in I$$

$$0 \leq y \leq 1$$

$$0 \leq w \leq 1$$

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$$\begin{aligned} \text{Minimize:} \quad & F(x) = \{f_1(x), f_2(x)\} \\ \text{Subject to:} \quad & x \in X \end{aligned}$$

- **Choose one objective:** f_1 or f_2? Let's select $f_1(x)$.
- **Restrict the other objective:** Introduce a constraint on the other objective:

$$\text{Minimize } f_1(x) \quad \text{subject to: } f_2(x) \leq \epsilon \text{ and } x \in X.$$

- By varying the ϵ values, the method generates different trade-off solutions.

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Notation

$$
\begin{aligned}
& \text{Minimize: } y \\
& \text{Minimize: } w \\
\text{subject to: } & \sum_{i \in I} D_{ij}\, x_i + A_j\, y \geq A_j,\ \forall j \in J \\
& \sum_{i \in I} L_{ijk}\, x_i + B_{jk}\, w \geq B_{jk},\ \forall j \in J,\ \forall k \in \{1, \ldots, n_j\} \\
& \sum_{i \in I} c_i\, x_i \leq B_c \quad \forall i \in I \\
& 0 \leq y \leq 1 \\
& 0 \leq w \leq 1 \\
& x_i \in \{0, 1\} \quad \forall i \in I
\end{aligned}
$$

$\mathcal{L}$: List of feasible nondominated solutions:

$$\mathcal{L} = \{(x, y, w)\}$$

N_ϵ: The total number of different epsilon values, indexed by n_ϵ

The extreme points of the Pareto front

(x^1, y^*, w^1)

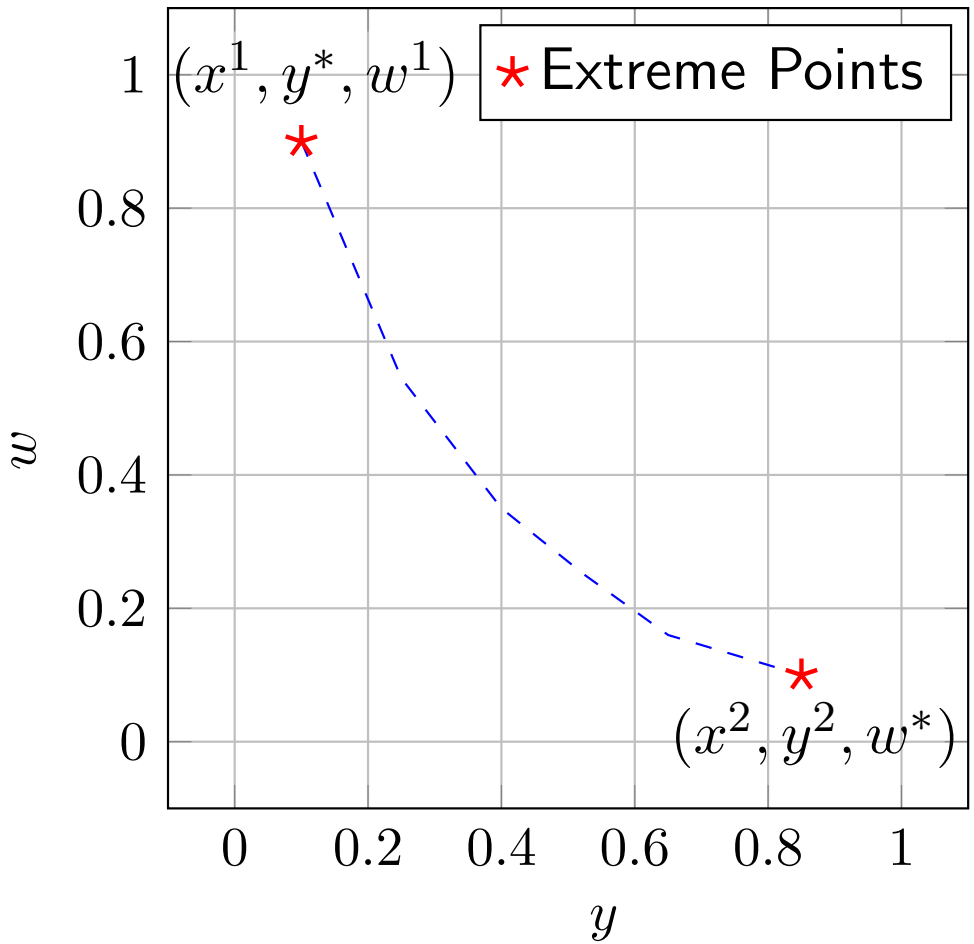
$y^* \leftarrow$ The smallest possible y

$w^1 \leftarrow$ Break ties by smaller w

(x^2, y^2, w^*)

$w^* \leftarrow$ The smallest possible w

$y^2 \leftarrow$ Break ties by smaller y



Epsilon-constraint subproblem

$Prob_1^{\epsilon}$: Minimize the first objective while treating the second objective as an epsilon constraint:

$$\text{Minimize: } y$$

$$\text{subject to: } \sum_{i \in I} D_{ij}\, x_i + A_j\, y \geq A_j, \quad \forall j \in J$$

$$\sum_{i \in I} L_{ijk}\, x_i + B_{jk}\, w \geq B_{jk}, \quad \forall j \in J,\ \forall k \in \{1, \ldots, n_j\}$$

$$\sum_{i \in I} c_i\, x_i \leq B_c, \quad \forall i \in I$$

$$x_i \in \{0, 1\} \quad \forall i \in I$$

$$0 \leq y \leq 1$$

$$0 \leq w \leq \epsilon$$

Epsilon-constraint subproblem

$Prob_2^{\epsilon}$: Minimize the second objective while treating the first objective as an epsilon constraint:

$$\text{Minimize: } w$$

$$\text{subject to: } \sum_{i \in I} D_{ij}\, x_i + A_j\, y \geq A_j, \quad \forall j \in J$$

$$\sum_{i \in I} L_{ijk}\, x_i + B_{jk}\, w \geq B_{jk}, \quad \forall j \in J,\ \forall k \in \{1, \ldots, n_j\}$$

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$$0 \leq w \leq 1$$

Algorithmic Framework

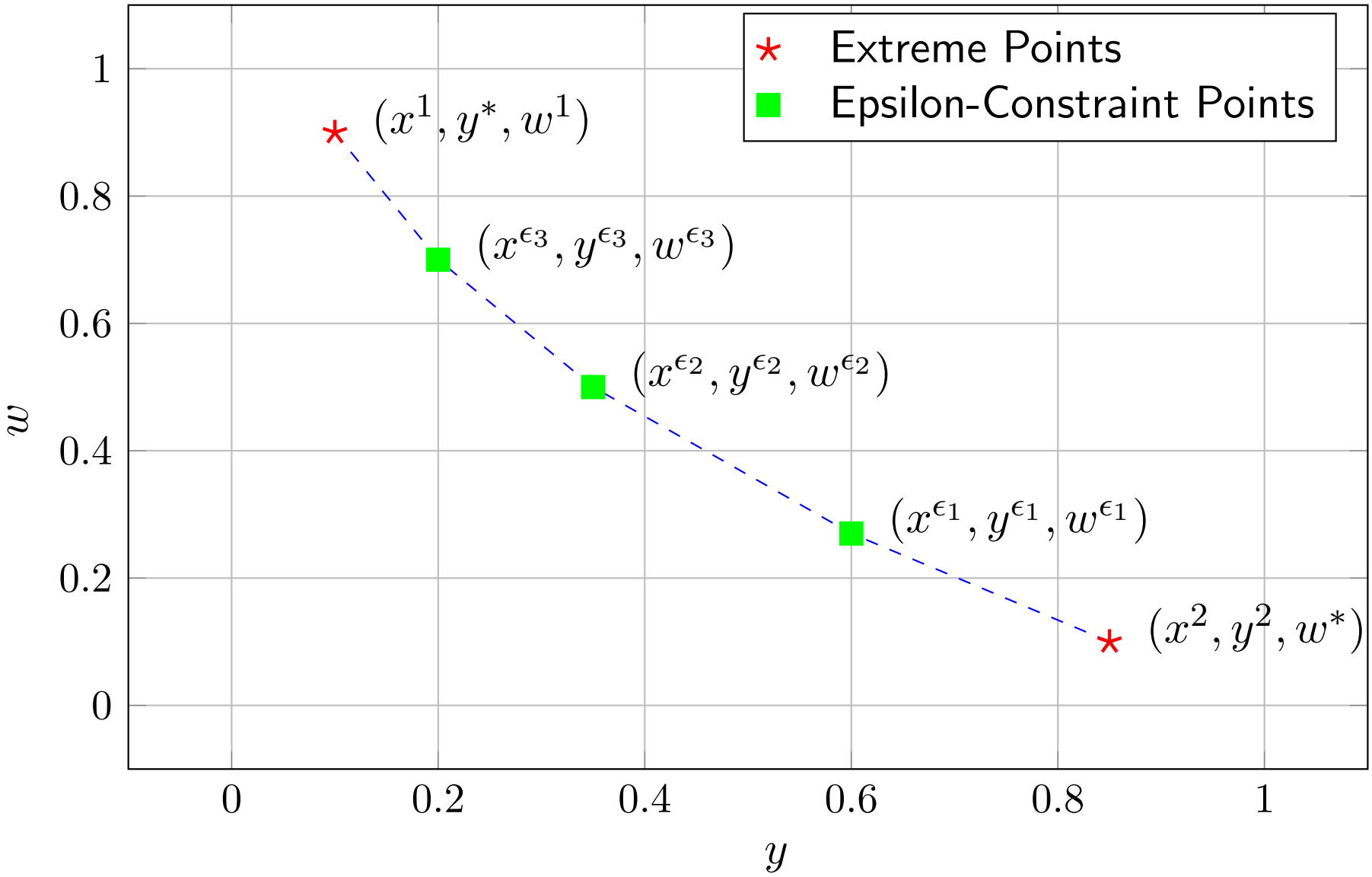
- The initial list of nondominated solutions: $\mathcal{L} \leftarrow \emptyset$
- Compute the extreme points (x^1, y^*, w^1) and (x^2, y^2, w^*), and add them to $\mathcal{L}$

Algorithmic Framework

- The initial list of nondominated solutions: $\mathcal{L} \leftarrow \emptyset$
- Compute the extreme points (x^1, y^*, w^1) and (x^2, y^2, w^*), and add them to $\mathcal{L}$
- For $n_\epsilon \leftarrow 1$ to N_ϵ do
  - $\epsilon \leftarrow w^* + n_\epsilon \times \dfrac{w^1 - w^*}{N_\epsilon + 1}$
  - $y^\epsilon \leftarrow$ Solve $\mathbf{Prob}_1^\epsilon$ to compute the minimum value of y
  - $w^\epsilon \leftarrow$ Solve $\mathbf{Prob}_2^{y^\epsilon}$ to break ties by smaller w
  - Add $(x^\epsilon, y^\epsilon, w^\epsilon)$ to $\mathcal{L}$ if it does not already exist in $\mathcal{L}$

Algorithmic Framework

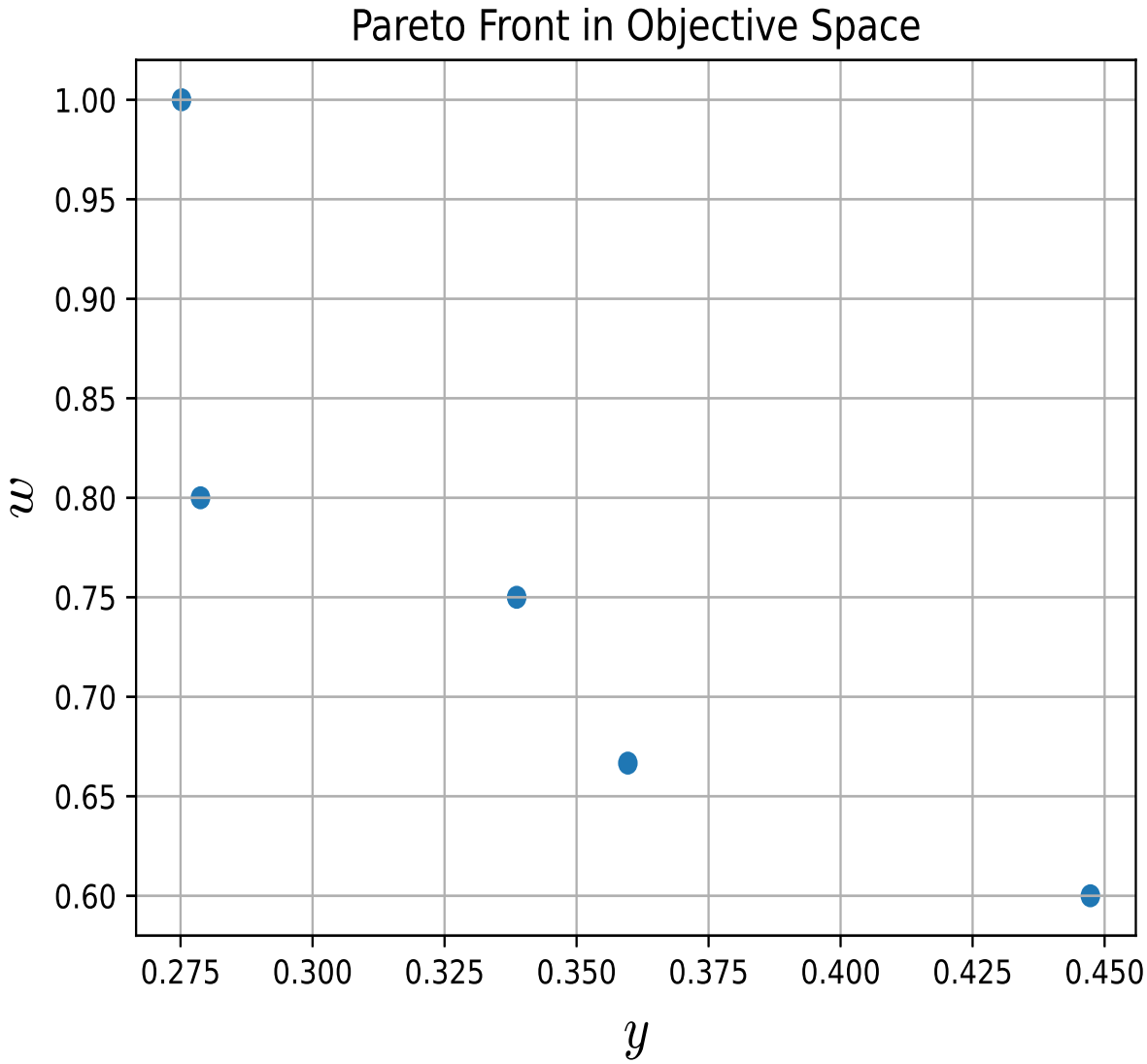
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- Return the final list $\mathcal{L}$


Extreme Points
Epsilon-Constraint Points
(x^1, y^*, w^1)
$(x^{\epsilon_3}, y^{\epsilon_3}, w^{\epsilon_3})$
$(x^{\epsilon_2}, y^{\epsilon_2}, w^{\epsilon_2})$
$(x^{\epsilon_1}, y^{\epsilon_1}, w^{\epsilon_1})$
(x^2, y^2, w^*)
1
0.8
0.6
0.4
0.2
0
w
0
0.2
0.4
0.6
0.8
1
y

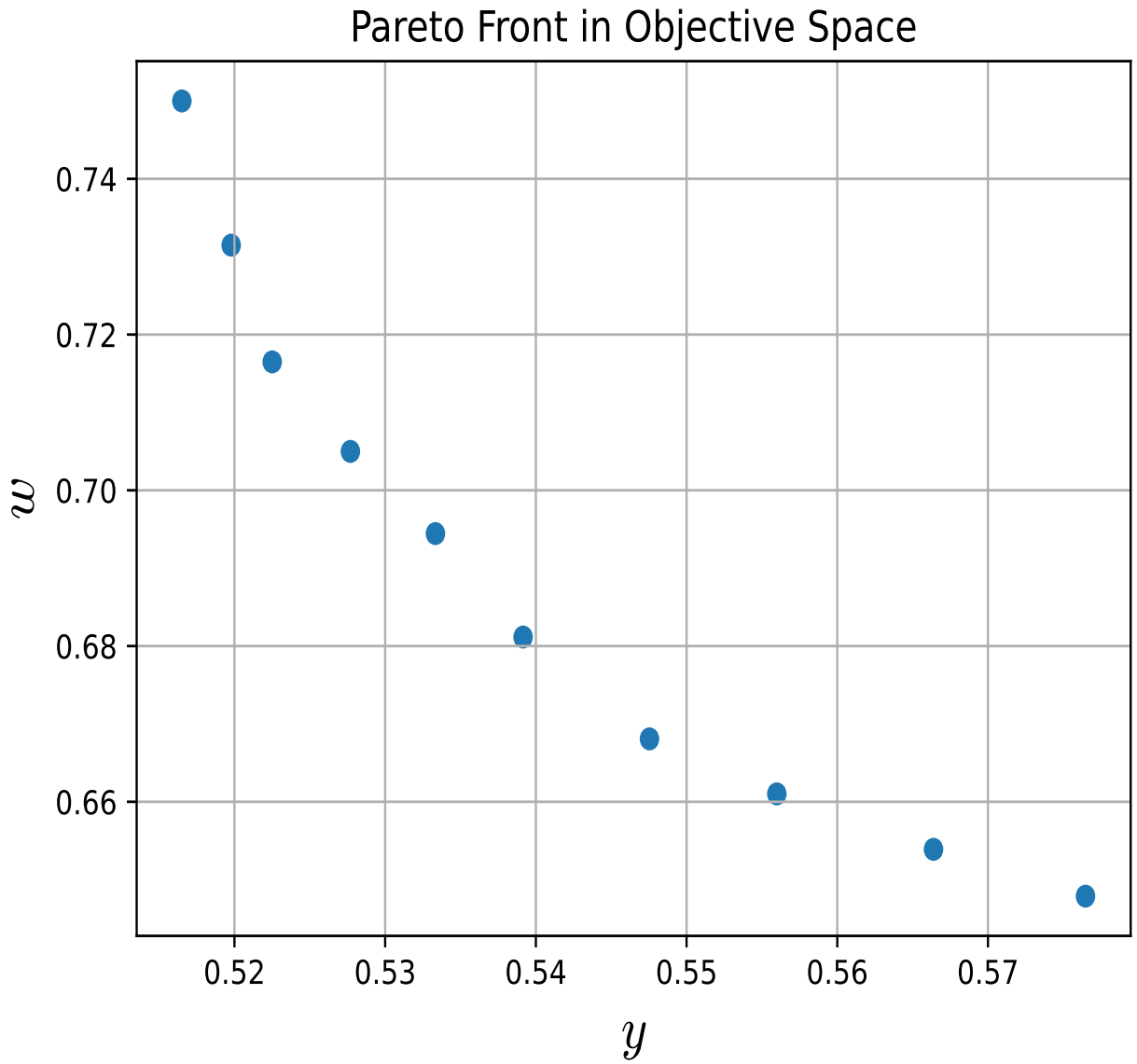
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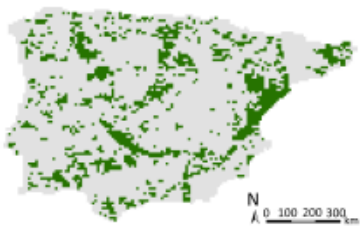
The approximation to the Pareto front, simulated dataset, 100 planning units



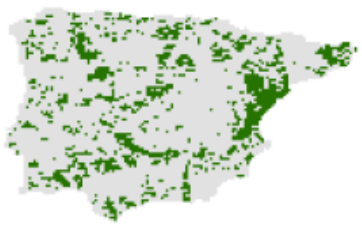
The approximation to the Pareto front, empirical dataset, Iberian case study, 7955 planning units



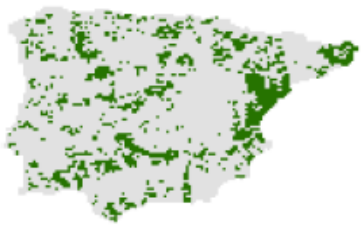
Spatial map of the selected planning units, where green units are selected and grey units are not



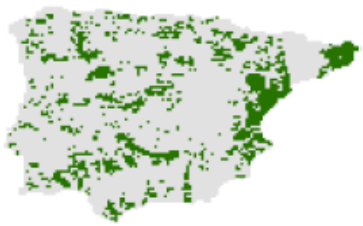
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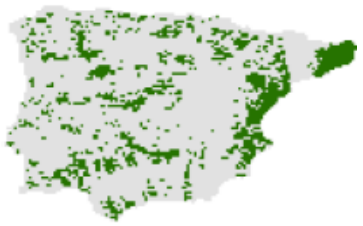
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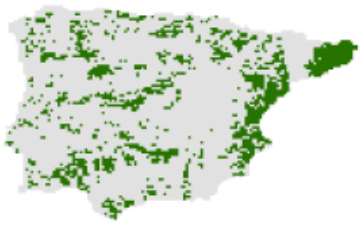
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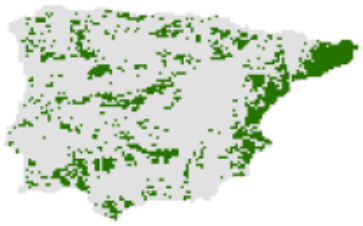
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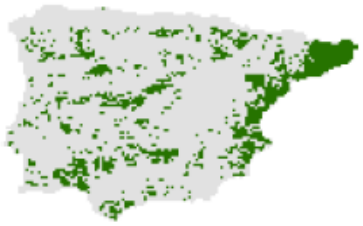
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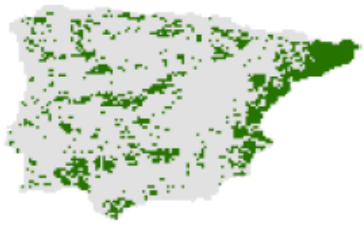
6



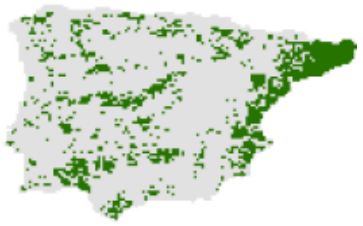
7



8



9



10

Thank you for your attention!

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